Proof. Let X be a continuous random variable with symmetric distrubution $f(x)$, with mean μ and variance σ^2. Observe that the distribution of a truncated variable is as follows: $g(x) = \frac{f(x)}{F(\mu-2\sigma)+1-F(\mu+2\sigma)}$ when $x \geq \mu+2\sigma$ or $x \leq \mu-2\sigma$, $g(y) = 0$ otherwise. Observe that

$$E[X|x \geq \mu + 2\sigma \text{ or } x \leq \mu - 2\sigma] = \int_{-\infty}^{\infty} g(x)xdx$$

$$= \int_{-\infty}^{\mu-2\sigma} g(x)xdx + \int_{\mu-2\sigma}^{\mu+2\sigma} g(x)xdx + \int_{\mu+2\sigma}^{\infty} g(x)xdx$$

By symmetry of $f(\cdot)$ we know that $F(\mu - 2\sigma) = 1 - F(\mu + 2\sigma)$ which implies that $\frac{f(x)}{F(\mu-2\sigma)+1-F(\mu+2\sigma)} = \frac{f(x)}{2F(\mu-2\sigma)}$, thus

$$E[X|\cdot] = \int_{-\infty}^{\mu-2\sigma} \frac{f(x)}{2F(\mu-2\sigma)}xdx + \int_{\mu-2\sigma}^{\mu+2\sigma} 0xdx + \int_{\mu+2\sigma}^{\infty} \frac{f(x)}{2F(\mu-2\sigma)}xdx$$

$$= \frac{1}{2F(\mu-2\sigma)} \left[\int_{-\infty}^{\mu-2\sigma} f(x)xdx + \int_{\mu+2\sigma}^{\infty} f(x)xdx\right]$$

$$= \frac{1}{2F(\mu-2\sigma)} \left[\int_{-\infty}^{\mu-2\sigma} f(x)xdx + \int_{\mu+2\sigma}^{\infty} f(x)xdx + \int_{\mu-2\sigma}^{\mu+2\sigma} f(x)xdx - \int_{\mu-2\sigma}^{\mu+2\sigma} f(x)xdx\right]$$

$$= \frac{1}{2F(\mu-2\sigma)} \left[\mu - \int_{\mu-2\sigma}^{\mu+2\sigma} f(x)xdx\right]$$

$$= \frac{1}{2F(\mu-2\sigma)} \left[\mu - (F(\mu+2\sigma) - F(\mu-2\sigma)) \int_{\mu-2\sigma}^{\mu+2\sigma} \frac{f(x)}{F(\mu+2\sigma) - F(\mu-2\sigma)}xdx\right]$$

It has been shown elsewhere that $\int_{\mu-2\sigma}^{\mu+2\sigma} \frac{f(x)}{F(\mu+2\sigma)-F(\mu-2\sigma)}xdx = \mu$, thus

$$= \frac{1}{2F(\mu-2\sigma)} [\mu - (F(\mu+2\sigma) - F(\mu-2\sigma))\mu] = \frac{1}{2F(\mu-2\sigma)} [1 - (1 - 2F(\mu-2\sigma))] \mu$$

since $F(\mu+2\sigma) = 1-F(\mu-2\sigma)$ we know $F(\mu+2\sigma)-F(\mu-2\sigma) = 1-2F(\mu-2\sigma)$, thus

$$E[X|x \geq \mu + 2\sigma \text{ or } x \leq \mu - 2\sigma] = \frac{1}{2F(\mu-2\sigma)} [2F(\mu-2\sigma))] \mu = \mu$$

□